1. B $(x+2)^2+(y-8)^2=68 \rightarrow A=68, h=-2, k=8 \rightarrow A+h+k=74$

2. C The point (1,1) is on the first line. Thus, using the equation for the distance from a point to the second line gives $\frac{|6+4-20|}{\sqrt{6^2+4^2}}=\frac{5}{\sqrt{13}}$

3. C $\frac{(x-2)^2}{9}-\frac{(y-3)^2}{16}=1 \rightarrow \text{center}=(2,3), a=3, b=4 \rightarrow \text{slope}=\pm\frac{b}{a}=\pm\frac{4}{3}$. Thus, the two possible asymptotes occur at $y-3=\pm\frac{4}{3}(x-2) \rightarrow \pm\frac{3}{4}(y-3)=x-2 \rightarrow$ $\frac{3}{4}y-\frac{9}{4}=x-2$ or $-\frac{3}{4}y+\frac{9}{4}=x-2$. Choosing the latter gives $x+\frac{3}{4}y-\frac{17}{4}=0 \rightarrow$ $b=\frac{3}{4}, c=-\frac{17}{4} \rightarrow \frac{b}{c}=-\frac{3}{17}$

4. C $\frac{(x-1)^2}{3}+\frac{(y-8)^2}{9}=1 \rightarrow a^2=9, b^2=3, c=\sqrt{a^2-b^2}=\sqrt{6}$. The center is at (1,8) and the directrices occur at $\frac{a^2}{c}$ to the top and bottom of the ellipse, thus occuring at $y=8-\frac{9}{\sqrt{6}}, 8+\frac{9}{\sqrt{6}}$. Thus the product yields $\frac{101}{2}$.

5. B Rewrite the equation to $(y+4)^2=4*\frac{3}{2}(x+2)$. This means that the length of the latus rectum is 6 and the focus occurs at $(-2+\frac{3}{2},-4)$ since the parabola is facing the right. This means $a=6, b=-\frac{1}{2}, c=-4 \rightarrow abc=12$

6. B $\frac{(x-1)^2}{3}+\frac{(y-8)^2}{9}=1 \rightarrow a^2=9, b^2=3, c=\sqrt{a^2-b^2}=\sqrt{6}$. Latus rectum length is $\frac{2b^2}{a}=2$ and the length of the other side of the rectangle is $2c=2\sqrt{6}$. Thus, the area of the rectangle is $4\sqrt{6}$ and the area of the ellipse is $ab\pi=3\sqrt{3}\pi$. The answer is $3\sqrt{3}\pi-4\sqrt{6}$.

7. E The difference in circumference is 2018 times. Because you are revolving about another circle, add one to 2018 for the total number of revolutions to get 2019. (Also, if you're convinced that the answer is 2018, think about two congruent circles. If you revolve one circle about the other one, the answer you get is 2.)

8. B I is false trivially. II is false since the eccentricity should be $\sqrt{2}$. III is true trivially. IV is never true trivially. V is true from $xy=1$. Thus, 3 choices are true.

9. A There are 1009 chords from the points that are diameters. From each chord, the third point will always form a right triangle due to the 180° subtended arc so you have 2016 points to choose from. Thus, the answer is $\frac{1009*2016}{\binom{2018}{3}}=\frac{3}{2017}$. 3+2017=2020.

10. C Instead of trying to find the number of acute triangles, let's find the number of obtuse triangles. There are 1008 sets of 2018 chords with equal lengths that are not the diameter. For any chosen chord, all points in the smaller subtended arc of the chord will form an obtuse triangle with the chord. Thus, the number of obtuse triangles is equal to $2018\sum_{n=0}^{1007} n$. Thus, the probability to make an obtuse triangle is $\frac{2018*1008*1007}{2*\binom{2018}{3}}=\frac{3021}{4034}$. From question 9, the probability to make a right triangle is $\frac{3}{2017}$. Thus, the probability to make an acute triangle is $1-\frac{3021}{4034}-\frac{3}{2017}=\frac{1007}{4034}$. $a+b=5041$

11. E The graph is a hyperbola centered at the origin which means that the directrices are parallel and that the y-intercepts are opposites of each other. Thus, the difference of the slopes and the sum of the y-intercepts are both 0. Thus, the answer is 0.

12. B $e = \frac{f}{d}$ where f is the distance from a point on the conic to the focus while d is the distance from that point to the directrix. $f = \sqrt{1+1} = \sqrt{2}, d = \frac{|1+1-6|}{\sqrt{2}} \to \frac{f}{d} = \frac{1}{2}$.

13. B Let (x, y) represent the points on the conic. Thus, those points must meet the criteria for the eccentricity so $f = \sqrt{x^2+y^2}, d = \frac{|x+y-6|}{\sqrt{2}} \to \frac{f}{d} = \frac{1}{2} \to 2f = d \to 4f^2 = d^2 \to 4x^2 + 4y^2 = \frac{(x+y-6)^2}{2} \to 8x^2 + 8y^2 = x^2 + 2xy + y^2 - 12x - 12y + 36$ so rewriting gives $7x^2 - 2xy + 7y^2 + 12x + 12y - 36 = 0$. The sum is 2.

14. B I is 0 because it is a circle. II has $b = 2, a = 4, c = \sqrt{20} \to e = \frac{c}{a} = \sqrt{\frac{20}{16}} > \sqrt{2}$. III is a parabola so it has eccentricity of 1. IV is a rectangular hyperbola so it has eccentricity of $\sqrt{2}$. V is an ellipse so the eccentricity is between 0 and 1. This gives IV, II, III, V, I.

15. C Simply, half of the area of the ellipse $4x^2 + y^2 = 9 \to \frac{x^2}{\left(\frac{3}{2}\right)^2} + \frac{y^2}{3^2} = 1$. Thus the area is $\frac{ab\pi}{2} = \frac{9\pi}{4}$.

16. B $a = 2, b = 4$, latus rectum $= \frac{2b^2}{a} = 16$

17. C Sketching the region, we see that it is a triangle below the y-axis. Start by choosing a x_1 value on the line $y = -3x + 1$. The corresponding x value for the second line is solved through $-3x_1 + 1 = x - 3$, yielding $x = -3x_1 + 4$. Thus, the length of our rectangle is $-3x_1 + 4 - x_1 = -4x_1 + 4$, and the height of our rectangle is $-(-3x_1 + 1)$, because the y value is negative. The area is $-(-3x_1 + 1)(-4x_1 + 4) = -12x^2 + 16x - 4$. This is a parabola, which has its vertex at $-\frac{b}{2a} = \frac{2}{3}$. Plugging that in gives us $\frac{4}{3}$ as the maximum.

18. A Multiplying out the matrix gives us the following system of equations: $a_1x^2 = 2x^2$, $2a_2xy = 0, 2a_3x = 8x, a_4y^2 = 6y^2, 2a_5y = 2y, a_6 = -4$. This gives us the matrix: $\begin{bmatrix} 2 & 0 & 4 \\ 0 & 6 & 1 \\ 4 & 1 & -4 \end{bmatrix}$ which has a determinant of $2 \cdot 6 \cdot -4 - 4 \cdot 6 \cdot 4 - 2 = -146$.

19. C $a^2 = 16, b^2 = 20, c^2 = a^2 + b^2 = 36 \to c = 6$. The center is located at $(-2,1)$ so the foci are located at $(-8,1)$ and $(4,1)$. We are only interested in the first focus. The length of the latus rectum is $\frac{2b^2}{a} = 10$. Thus, we can have two parabola possibilities at $(y-1)^2 = 10\left(x + \frac{10}{4} + 8\right), (y-1)^2 = -10\left(x - \frac{10}{4} + 8\right)$. Setting $y = 0 \to x = -\frac{52}{5}, -\frac{28}{5}$. Thus, the product gives $\frac{1456}{25}$.

20. A Instead of trying to factor the equation, we can work this problem backwards. We know that $(mx + b - y)(nx + c - y) = bc + bnx - by + cmx - cy + mnx^2 - mxy - nxy + y^2 = \frac{-x^2+4xy-3y^2-3x+7y+constant}{-3}$ to keep the coefficients of y the same. This gives us the system of equations: $-b - c = -\frac{7}{3}, bn + cm = 1, mn = \frac{1}{3}, -m - n = -\frac{4}{3}$. Solving the last two equations gives us $m = 1, n = \frac{1}{3}$ or vice versa, it doesn't matter, just keep it consistent when solving for *b, c.* Solving the first two equations gives us $b = 2, c = \frac{1}{3}$. So $bm + cn = \frac{19}{9}$

21. D Referring to the expansion given at the front of the test, we have $f + ah^2 + bhk + ck^2 - 2ahx - bkx + ax^2 - bhy - 2cky + bxy + cy^2 = x^2 + xy + y^2 + 2x - 8y + 4$. We use f instead of 1 to account for multiplying the conic equation by a nonzero constant. This gives us the equations" $a = 1, c = 1, -2ah - bk = 2, -bh - 2ck = -8$, $-f + ah^2 + bhk + ck^2 = 4$. Solving gives $a = 1, b = 1, c = 1, h = -4, k = 6, f = 24$. Thus, the center is $(h, k) = (-4,6)$.

22. A Referring to the formula, $\frac{2\pi i}{\sqrt{b^2-4ac}}$ is the area when $f = 1$. Thus, when $f \neq 1$, the formula becomes $\frac{2\pi i}{\frac{1}{f}\sqrt{b^2-4ac}}$ since a, b, c will be scaled down a factor of f. Thus, $\frac{2\pi i}{\frac{1}{f}\sqrt{b^2-4ac}} = \frac{2\pi i}{\frac{1}{24}\sqrt{1-4}}$ $= 16\pi\sqrt{3}$

23. C First, we must find the equation of the directrix. Since the focus is at $(3,3)$ and the vertex is at $(-1,1)$, then the directrix will contain a point the same distance opposite from the vertex to the focus. The directrix will also be perpendicular to the line connecting the focus and vertex. Thus, the directrix will contain $(-5,-1)$ and have a slope of -2 giving $-2(x+5) = y+1 \rightarrow 2x + y + 11 = 0$. Thus, we set up the parabola to contain the point (x, y), which is equidistant to the focus and parabola giving $\sqrt{(x-3)^2 + (y-3)^2} = \frac{|2x+y+11|}{\sqrt{5}} \rightarrow$ squaring both sides and multiplying by 5 $\rightarrow$ $5x^2 + 5y^2 - 30x - 30y + 90 = 4x^2 + 4xy + 44x + y^2 + 22y + 121 \rightarrow$ $x^2 - 4xy + 4y^2 - 74x - 52y - 31 = 0$. Add the coefficients = -156. Answer = 156

24. A The equation for the ellipse can be written as $\frac{(x-2)^2}{8} + \frac{(y+3)^2}{12} = 1$ and for the hyperbola $(x-2)(y-3) = k-6$. Now let $x_0 = x-2, y_0 = y+3, p = 2\sqrt{2}, q = 2\sqrt{3}$. Then the equations of the ellipse and hyperbola are respectively transformed into $\frac{x_0^2}{p^2} + \frac{y_0^2}{q^2} = 1$ and $x_0y_0 = k-6$. A necessary condition for the ellipse and hyperbola to be tangent is that they have one point of intersection for some $x_0 > 0, y_0 > 0$. This point of intersection will occur when $y_0 = \frac{k-6}{x_0}$. Substituting this back into the transformed ellipse gives $\frac{x_0^2}{p^2} + \frac{(k-6)^2}{x_0^2q^2} = 1 \rightarrow q^2x_0^4 - p^2q^2x_0^2 + p^2(k-6)^2 = 0$, which is a quadratic in x_0^2. Since we want a single point of intersection, the discriminant of this quadratic must be 0 giving $p^4q^4 - 4p^2q^2(k-6)^2 = 0 \rightarrow p^2q^2 - 4(k-6)^2 = 0 \rightarrow k-6 = \pm\frac{pq}{2}$ so $k = 6 + 2\sqrt{6}. abc = 72$

25. C In order to find a perpendicular bisector, we find the midpoint between two points and the reciprocal inverse of the slope of the two points. Thus, the perpendicular bisector of $A, B \rightarrow -\frac{1}{4}(x-2) = y-5$ and $A, C \rightarrow \frac{1}{2}x = y-3$. The intersection of these lines is the center of the circle which is $(\frac{10}{3}, \frac{14}{3})$. The radius is determined from the center to any of the three points, say A, giving us $\sqrt{\frac{170}{9}}$. Thus, $hk + r^2 = \frac{310}{9}$.

26. B Obviously, this is an ellipse. We know that $a = 10, c^2 = a^2 - b^2, Area = ab\pi$. We need to minimize c to maximize b. From the information we are given, we know that $20 = \sqrt{5^2 + 12^2} + \sqrt{p^2 + q^2} \rightarrow 49 = p^2 + q^2$. This means the other focus lies on the circle $x^2 + y^2 = 7^2$. Thus, to minimize the focal length c, the other point must be

collinear to (0,0) and (5,12). Thus, we see that the distance from (5,12) to (0,0) is 13, (p,q) to (0,0) is 7, so $2c = 6 \to c = 3 \to b = \sqrt{91}$ giving area of $10\pi\sqrt{91}$.

27. E The "smallest" area would be 0 since the locus of points would be a line segment collinear to (5,12) and (0,0) and lines on the previous circle but in the 3rd quadrant.

28. D The line through P, Q is perpendicular to m which is expressed algebraically by the equation $b(x' - x_1) = a(y' - y)$. Also, $\left(\frac{x_1+x'}{2}, \frac{y_1+y'}{2}\right)$ is the midpoint of $\overline{PQ}$ and is on line m. This geometric fact is expressed algebraically by the equation $a\left(\frac{x_1+x'}{2}\right) + b\left(\frac{y_1+y'}{2}\right) + c = 0$. Rewriting these two previous equations gives us $\begin{cases} bx' - ay' = bx_1 - ay_1 \\ ax' + by' = -2c - ax_1 - by_1 \end{cases} \to \begin{cases} b^2x' - aby' = b^2x_1 - aby_1 \\ a^2x' + aby' = -2ac - a^2x_1 - aby_1 \end{cases}$. Solving this gives us $x' = \frac{b^2x_1 - a^2x_1 - 2aby_1 - 2ac}{a^2+b^2} = x_1 - \frac{2a(ax_1+by_1+c)}{a^2+b^2}$. Solving for y' in a similar fashion gives us $y' = y_1 - \frac{2b(ax_1+by_1+c)}{a^2+b^2}$.

29. C Using the formulas we just derived, we get $x' = x - \frac{4(2x-y)}{5} = \frac{4y-3x}{5}, y' = y - \frac{-2(2x-y)}{5} = \frac{4x+3y}{5}$. Plugging this into $x'y' + x' = 1$ gives us $\frac{4y-3x}{5} \cdot \frac{4x+3y}{5} + \frac{4y-3x}{5} = 1 \to -12x^2 + 7xy - 15x + 12y^2 + 20y - 25 = 0$. The absolute value of the sum of the coefficients is 13.

30. B $x^2 + 2y^2 + 2x + 8y + 9 = 0 \to (x+1)^2 + 2(y+2)^2 = 0 \to$ the point $(-1,-2)$. Answer is $2018 - (-1) = 2019$.